Cooperative Spectrum Sharing via Controlled Amplify-and-Forward Relaying

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***Abstract*—We consider a spectrum sharing protocol in which a secondary system operates on the same spectrum as a primary system, without adversely affecting the rate of the primary system. The protocol comprises of a two-phase transmission. In the first phase, the primary transmitter transmits a primary signal to the primary receiver, which is also received at the secondary transmitter and receiver. The secondary transmitter amplifies the received primary signal and generates a linearly weighted combination of this signal and the secondary signal. The weight is a variable power allocation factor α ($0 \leq \alpha \leq 1$). This composite signal is then broadcast in the second transmission phase. We analyze the achievable rates for the primary and secondary systems, and determine α such that the rate of primary system with this spectrum sharing protocol is no worse than that in the absence of the secondary system. We show that the spectrum sharing protocol can improve the rate of primary system by an appropriate choice of α, while at the same time achieve secondary spectrum access.**

I. INTRODUCTION

The demand for radio spectrum has increased dramatically with the explosive growth of wireless services and applications. Spectrum has been traditionally allocated by exclusive licensing in order to limit the interference between different wireless systems. This approach has led to a scarcity of unallocated spectrum as evidenced in the radio spectrum allocation charts [1]. The fact that most spectrum is already licensed does not however mean that it is being utilized efficiently. Spectrum measurements [2] show that large portions of spectrum remain under-utilized. In this paper, we propose a spectrum sharing scheme in which secondary usage of spectrum is facilitated by controlled cooperative transmission by the secondary transmitter.

The spectrum sharing system under consideration in this paper consists of a primary system and a secondary system. The primary system, comprising a primary transmitter (PT) and primary receiver (PR), has licensed rights to operate in a certain portion of the spectrum. The secondary system, comprising a secondary transmitter (ST) and secondary receiver (SR), can operate on a secondary basis in this spectrum, with the constraint that its operation does not adversely affect the primary system performance.

Different models for dynamic spectrum sharing have been proposed in literature [3]–[8]. Spectrum sharing protocols under a centralized resource management entity called spectrum broker have been explored in [3]. In [7], [9], secondary spectrum use is facilitated by detecting primary signal transmissions and operating on spectrum portions where primary systems are determined to be silent. This detect-and-avoid mechanism ensures that secondary system operation does not cause harmful interference to primary systems. In [5], [6], information-theoretic results were presented by considering a scenario wherein the secondary transmitter applies dirty-paper coding [10] to pre-compensate the interference its transmission would cause to the primary receiver. This technique however requires the secondary transmitter to have non-causal information about the primary system transmission. References [4], [5], [6] implicitly point to the role cooperation could play in spectrum sharing protocols. We advance this view by considering a spectrum sharing protocol in which the secondary transmitter performs controlled cooperative transmissions.

The secondary system has to ensure that the achievable rate of primary system under spectrum sharing is no worse than that without spectrum sharing. The following spectrum sharing protocol is adopted to ensure this requirement. In the first transmission phase, the primary signal is transmitted by PT to PR, and is also received by ST and SR. The primary signal is amplified at ST to maintain its power constraint. Then a superimposed signal which is a linear weighted combination of the amplified primary signal and the secondary signal is generated by ST. The weight α ($0 \leq \alpha \leq 1$), is the power allocation factor representing a fraction of the total transmit power at ST that is allocated to primary signal. This weighted linear composite signal is then broadcast by ST in the second transmission phase. At PR, the primary signal is decoded based on the received signals in the two transmission phases. At SR, interference cancelation is applied to cancel out the primary signal component and retrieve the secondary signal.

We analyze the proposed protocol by deriving the achievable rates of the primary and secondary systems. We show that by a judicious choice of the factor, α, the secondary system can operate without adversely affecting the rate of primary system. By controlling α, the primary system rate can either

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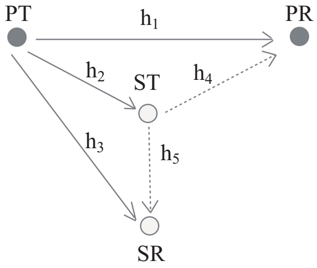

Fig. 1. Spectrum sharing system.

be maintained to be the same as without spectrum sharing, or can be improved by a desired margin. An improvement in the primary system rate can be achieved particularly when the direct link between PT and PR is highly attenuated, in scenarios for e.g., when PR is in a shadowing region with respect to PT.

In this paper, we use $\boldsymbol{E}\{\cdot\}$ to denote expectation. A circularly symmetric complex Gaussian random variable z with mean μ and variance σ^2 is denoted as $z \sim \mathcal{CN}(\mu, \sigma^2)$. An exponential distributed random variable x with parameter λ is denoted as $x \sim \mathcal{E}(\lambda)$ and the probability density function is given as

$$f_X(x) = \frac{1}{\lambda} e^{-\frac{x}{\lambda}} U(x)$$

where $U(x)$ denotes the unit step function. An $N \times N$ identity matrix is denoted by I_N. The transpose and conjugate transpose of a matrix $\boldsymbol{A}$ are denoted by $\boldsymbol{A}^{\mathrm{T}}$ and $\boldsymbol{A}^{\mathrm{H}}$ respectively.

II. PROTOCOL DESCRIPTION

The system configuration under consideration is shown in Fig. 1. The channel coefficients of links PT→PR, PT→ST, PT→SR, ST→PR, and ST→SR are denoted as h_1, h_2, h_3, h_4, and h_5 respectively. We assume Rayleigh flat fading channels with $h_i \sim \mathcal{CN}(0, \beta_i)$, $i = 1, 2, 3, 4, 5$. We also denote $\gamma_i = |h_i|^2$. Let x_p and x_s denote the primary and secondary signals respectively, where $\boldsymbol{E}\{x_p^* x_p\} = 1$ and $\boldsymbol{E}\{x_s^* x_s\} = 1$. The transmit power at PT and ST is denoted by P_p and P_s respectively.

A. Achievable rate for primary system

We first consider the situation where only the primary system is operating, i.e. there is no spectrum sharing. The primary signal is transmitted from PT to PR over channel h_1, with transmit power P_p. Thus, the achievable rate of the primary system is given as

$$R_n = \log_2 \left(1 + \frac{P_p \gamma_1}{\sigma^2}\right) \tag{1}$$

where σ^2 is the noise variance. As pointed out earlier, spectrum sharing with a secondary system is allowed under the condition that its operation does not cause the rate of primary system to fall below R_n.

We now describe the spectrum sharing protocol. The transmission is split into two phases. In the first transmission phase, as shown by the solid line in Fig. 1, the primary signal x_p is broadcast by PT. Denoting the signal received by PR, ST, and SR in the first transmission phase as y_{11}, y_{21}, and y_{31} respectively, we have

$$y_{j1} = \sqrt{P_p} h_j x_p + n_{j1} \tag{2}$$

where $j = 1, 2, 3$ and n_{j1} is the zero mean additive white Gaussian noise (AWGN) at the respective terminals in the first transmission phase with $\boldsymbol{E}\{n_{j1}^* n_{j1}\} = \sigma^2$.

After reception in the first transmission phase, ST normalizes the received signal based on its power constraint and further amplifies it with the power allocation factor α followed by adding in its own secondary signal x_s to generate a superimposed signal

$$y_{22} = g y_{21} + \sqrt{P_s(1-\alpha)} x_s \tag{3}$$

where $0 \leq \alpha \leq 1$, and the normalization factor g is given by

$$g = \sqrt{\frac{P_s \alpha}{P_p \gamma_2 + \sigma^2}}. \tag{4}$$

In the second transmission phase, as shown by the dotted lines in Fig. 1, the signal y_{22} is broadcast to both PR and SR. The signal received at PR is given by

$$\begin{aligned} y_{12} =& h_4 y_{22} + n_{12} \\ =& (\sqrt{P_p} g h_2 h_4) x_p + (\sqrt{P_s(1-\alpha)} h_4) x_s + g h_4 n_{21} + n_{12}, \end{aligned} \tag{5}$$

where n_{12} is the zero mean AWGN at PR in the second transmission phase and $\boldsymbol{E}\{n_{12}^* n_{12}\} = \sigma^2$. Signals y_{11} and y_{12} are combined at PR for the decoding of x_p. At PR the two-phase transmission of x_p can be written as an equivalent single-input-multiple-output (SIMO) channel,

$$\boldsymbol{Y} = \boldsymbol{H} x_p + \boldsymbol{N} \tag{6}$$

where $\boldsymbol{Y} = [y_{11} \;\; y_{12}]^{\mathrm{T}}$, $\boldsymbol{H} = \sqrt{P_p}[h_1 \;\; h_2 h_4 g]^{\mathrm{T}}$, and $\boldsymbol{N} = [n_{11} \;\; \sqrt{P_s(1-\alpha)} h_4 x_s + g h_4 n_{21} + n_{12}]^{\mathrm{T}}$. Performing prewhitening, we obtain

$$\tilde{\boldsymbol{Y}} = \begin{bmatrix} \frac{y_{11}}{\sqrt{\sigma^2}} \\ \frac{y_{12}}{\sqrt{\lambda}} \end{bmatrix} = \tilde{\boldsymbol{H}} x_p + \tilde{\boldsymbol{N}} \tag{7}$$

where $\tilde{\boldsymbol{H}} = \sqrt{P_p}[\frac{h_1}{\sqrt{\sigma^2}} \;\; \frac{h_2 h_4 g}{\sqrt{\lambda}}]^{\mathrm{T}}$, $\lambda = P_s(1-\alpha)\gamma_4 + g^2 \gamma_4 \sigma^2 + \sigma^2$, and $\boldsymbol{E}\{\tilde{\boldsymbol{N}} \tilde{\boldsymbol{N}}^{\mathrm{H}} | \boldsymbol{H}\} = \boldsymbol{I}_2$. The achievable rate for this 1×2 SIMO channel is given by

$$\begin{aligned} R_p =& \frac{1}{2} \log_2 \left(\det\left(\boldsymbol{I}_2 + \tilde{\boldsymbol{H}} \tilde{\boldsymbol{H}}^{\mathrm{H}}\right)\right) \\ =& \frac{1}{2} \log_2 \left(1 + \frac{P_p \gamma_1}{\sigma^2} + \frac{P_p \gamma_2 \gamma_4 g^2}{\lambda}\right) \end{aligned} \tag{8}$$

where the factor $\frac{1}{2}$ accounts for the fact that the transmission of x_p is split carried out over two phases.

To make sure the achievable rate for primary system with the proposed spectrum sharing protocol is greater or equal to the case without spectrum sharing, we need

$$R_p \geq R_n. \tag{9}$$

We need to determine the range of α for which the condition in (9) is satisfied. Substituting (8) and (1) into (9), and considering the approximation

$$g \approx \sqrt{\frac{P_s \alpha}{P_p \gamma_2}} \tag{10}$$

which holds when $P_p \gamma_2 \gg \sigma^2$, we obtain

$$\alpha_{\mathrm{t}} = \frac{P_s \gamma_4 + \sigma^2}{\frac{P_s \gamma_4}{\rho(\rho+1)} + P_s \gamma_4 - \frac{P_s \gamma_4}{P_p \gamma_2} \sigma^2}. \tag{11}$$

where α_{t} is the minimum value for α such that (9) holds with equality, and $\rho = \frac{P_p \gamma_1}{\sigma^2}$. Note that due to the random nature of wireless channels, there is a non-zero probability that $\alpha_{\mathrm{t}} > 1$, so we let

$$\alpha^* = \min(\alpha_{\mathrm{t}}, 1). \tag{12}$$

As long as we choose a value of α that is greater than α^*, the rate of primary system can be improved. The achievable rate for the primary system under the proposed protocol will be greater than that in the case without spectrum sharing when $\alpha_{\mathrm{t}} < 1$ and $\alpha^* < \alpha \leq 1$.

From (11) and (12), we can obtain the value for α^* given the instantaneous values of h_1, h_2, h_4, P_p, and P_s. However, for a practical system, these instantaneous value may be difficult to obtain, for example, it is difficult for ST to obtain h_4 if there is no feedback link from PR to ST. In view of this problem, it is desirable to determine the power allocation factor without the knowledge of instantaneous channel realizations. To this end, we obtain an average lower bound for α_{t}.

Note that in (11), with $\frac{P_s \gamma_4}{\sigma^2} \gg 1$ and $\frac{P_p \gamma_2}{\sigma^2} \gg 1$, we obtain the following lower bound for α_{t},

$$\alpha^*_{\mathrm{lb}} = \frac{\rho(\rho+1)}{\rho^2 + \rho + 1}. \tag{13}$$

In order to apply α^*_{lb}, ST still needs to obtain the instantaneous value of γ_1, which might be difficult to obtain when there is no feedback link from PR to ST.

We now consider the average value of α^*_{lb}. From (13), we have

$$\begin{aligned} \boldsymbol{E}\{\alpha^*_{\mathrm{lb}}\} &= 1 - \boldsymbol{E}\{\frac{1}{\rho^2+\rho+1}\} \\ &= 1 - \frac{1}{\rho_1 - \rho_2} \boldsymbol{E}\{\frac{1}{\rho - \rho_1} - \frac{1}{\rho - \rho_2}\}, \end{aligned} \tag{14}$$

where $\rho_1 = \frac{-1+\sqrt{3}i}{2}$ and $\rho_2 = \frac{-1-\sqrt{3}i}{2}$. Since $\gamma_1 \sim \mathcal{E}\{\beta_1\}$, we have

$$\begin{aligned} \boldsymbol{E}\{\frac{1}{\rho - \rho_1}\} &= \frac{\sigma^2}{P_p} \int_0^\infty \frac{\frac{1}{\beta_1} e^{-\frac{\gamma_1}{\beta_1}}}{\gamma_1 - \frac{\sigma^2}{P_p}\rho_1} d\gamma_1 \\ &= \tau e^{-\tau \rho_1} E_1(-\tau \rho_1) \end{aligned} \tag{15}$$

where $\tau = \frac{\sigma^2}{P_p \beta_1} = 1/\boldsymbol{E}\{\rho\}$ and $E_1(\cdot)$ is the exponential integral [11] defined as

$$E_1(x) = \int_x^\infty \frac{e^{-t}}{t} dt$$

where $|\arg(x)| < \pi$. Similarly,

$$\boldsymbol{E}\{\frac{1}{\rho - \rho_2}\} = \tau e^{-\tau \rho_2} E_1(-\tau \rho_2). \tag{16}$$

Substituting (15) and (16) into (14), we have

$$\boldsymbol{E}\{\alpha^*_{\mathrm{lb}}\} = 1 - \frac{\tau e^{-\tau \rho_1} E_1(-\tau \rho_1) - \tau e^{-\tau \rho_2} E_1(-\tau \rho_2)}{\rho_1 - \rho_2} \tag{17}$$

B. Achievable rate for secondary system

For convenience, we write the signal received at SR in first transmission phase again as,

$$y_{31} = \sqrt{P_p} h_3 x_p + n_{31}. \tag{18}$$

The achievable rate for the PT→SR link in the first transmission phase is given by

$$R_{s1} = \frac{1}{2} \log_2 \left(1 + \frac{P_p \gamma_3}{\sigma^2}\right). \tag{19}$$

The signal received at SR in the second transmission phase is given as

$$\begin{aligned} y_{32} &= h_2 y_{22} + n_{32} \\ &= (\sqrt{P_p} g h_2 h_5) x_p + (\sqrt{P_s(1-\alpha)} h_5) x_s + g h_5 n_{21} + n_{32}. \end{aligned} \tag{20}$$

where n_{32} is the AWGN at SR in the second transmission phase and $\boldsymbol{E}\{n^*_{32} n_{32}\} = \sigma^2$.

Assuming SR is able to decode x_p successfully from (18), the interference component $(\sqrt{P_p} g h_2 h_5) x_p$ can be canceled out from y_{32} perfectly to obtain

$$y'_{32} = (\sqrt{P_s(1-\alpha)} h_5) x_s + g h_5 n_{21} + n_{32}. \tag{21}$$

Note that only the product $\sqrt{P_p} g h_2 h_5$ is needed and can be obtained in practice through the use of training symbols; knowledge of the individual channel coefficients h_2 and h_5 is not required.

The achievable rate for ST→SR link in the second transmission phase, given that the decoding of x_p at SR in the first transmission phase is successful, can be given as

$$R_{s2} = \frac{1}{2} \log_2 \left(1 + \frac{P_p (1-\alpha) \gamma_2 \gamma_5}{\alpha \gamma_5 \sigma^2 + \frac{P_p}{P_s} \gamma_2 \sigma^2}\right). \tag{22}$$

Since the achievable rate of the secondary transmission depends on the successful decoding of the primary signal at SR, the achievable rate of PT→SR link becomes the limit of the achievable rate for secondary transmission when $R_{s1} < R_{s2}$. Thus, the achievable rate for the secondary system is given by

$$R_s = \min\{R_{s1}, R_{s2}\}. \tag{23}$$

Comparing (8) and (23), we can observe that when α is increased, more power is allocated by ST for assisting (relaying the primary signal) the primary system and less power is used for the secondary system's own transmission which causes a respective increase and decrease of the achievable rate for primary and secondary systems. From (13), it is obvious that when $\rho \gg 1$, we have $\alpha^*_{\mathrm{lb}} \approx 1$ which indicates that in order

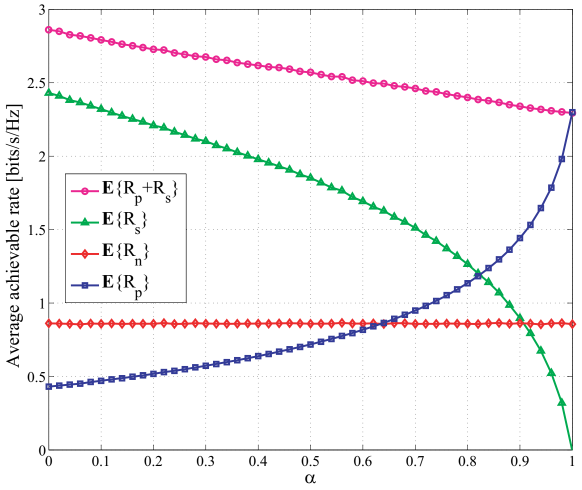

Fig. 2. Average achievable rate comparison.

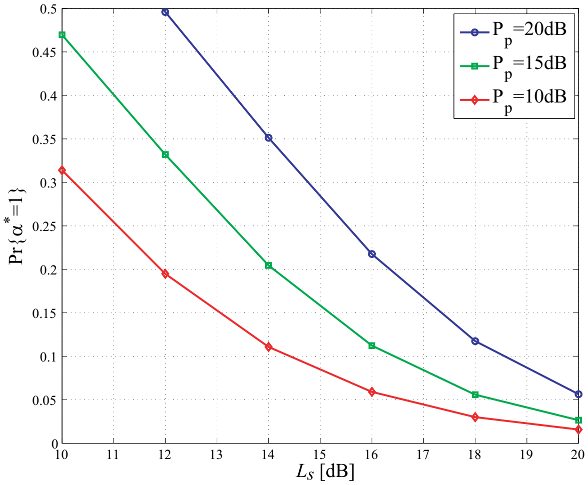

Fig. 3. Probability of $\Pr\{\alpha^* = 1\}$.

to maintain the same performance for primary system as the case without spectrum sharing, ST has to allocate almost all of its power to relay the transmission of primary system which will cause $R_s \approx 0$. In this case, the proposed spectrum sharing scheme will reduce to a conventional relaying system where ST purely plays a role of a relay. However, when the PT⟶PR link is weak due to deep fading or strong shadowing, i.e. γ_1 is very small compared to γ_i, $i = 2, 3, 4, 5$, we obtain a small value of ρ, in turn resulting in a small value of α^*. In this case, ST can choose an appropriate α value in the region $\alpha^* < \alpha \leq 1$ which provides significant performance improvement for the primary system while still achieving a reasonable rate for the secondary system.

III. SIMULATION RESULTS AND DISCUSSIONS

We assume that the direct link from PT to PR experiences an extra path loss and/or shadowing of L_sdB as compared to other links. We assume $\beta_i = 1, i = 2, 3, 4, 5$, $L_s = 20$dB, hence $\beta_1 = 10^{-2}$. We also assume $\frac{P_p}{\sigma^2} = \frac{P_s}{\sigma^2} = 20$dB. We first consider the achievable rate of primary and secondary systems as the power allocation factor α at ST is varied. In Fig. 2, we show the average achievable rates for the primary and secondary system with and without the proposed spectrum sharing protocol.

From Fig. 2, it is obvious that in the range $0.64 < \alpha < 1$, we have $\boldsymbol{E}\{R_p\} > \boldsymbol{E}\{R_n\}$ and $\boldsymbol{E}\{R_s\} > 0$, which indicates that the secondary system obtains transmission access while at the same time the average achievable rate for primary system is improved. Thus both the primary and secondary system achieve some performance gains. Furthermore, we can also observe that the average achievable sum rate under the proposed scheme is much larger than that of the case without spectrum sharing for all values of α.

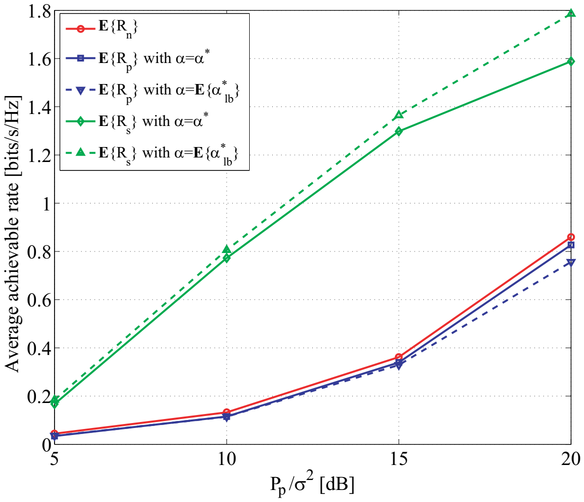

Fig. 4. Average achievable rate of primary and secondary systems.

As shown in Section II, in order to achieve performance gains for both primary and secondary systems, α should be larger than the threshold value α^*. When $\alpha^* = 1$, it is not possible to meet the requirement $R_p > R_n$. However, due to the random nature of wireless channels, $\Pr\{\alpha^* = 1\}$ has a non-zero value. In Fig. 3, we show $\Pr\{\alpha^* = 1\}$ with different values of L_s. We again assume $\frac{P_p}{\sigma^2} = \frac{P_s}{\sigma^2} = 20$dB, and $\beta_i = 1, i = 2, 3, 4, 5$. From Fig. 3, we can observe that with a weak PT⟶PR link, i.e. when L_s is large, $\Pr\{\alpha^* = 1\}$ is small. For example, when $L_s = 20$dB, $\Pr\{\alpha^* = 1\}$ is smaller than 5% for all three values of P_p, which indicates that for more than 95% of the channel realizations, in region where $\alpha^* < \alpha \leq 1$, we obtain performance gains for both the primary and secondary systems.

In Fig. 4, we show the average achievable rate for primary and secondary systems with and without the proposed spectrum sharing protocol. Again, we assume $\frac{P_p}{\sigma^2} = \frac{P_s}{\sigma^2}$, $\beta_1 = 10^{-2}$ and $\beta_i = 1, i = 2, 3, 4, 5$. We consider two choices of α: $\alpha = \alpha^*$ which is given in (12) and $\alpha = \boldsymbol{E}\{\alpha^*_{\text{lb}}\}$ which is given in (17). Note that for the first case, ST needs to obtain instantaneous values of h_1, h_2, h_4, P_p, and P_s to calculate α^*. However, for the second case, only $\boldsymbol{E}\{\rho\}$ is needed, which is the average SNR of PT→PR link and can be possibly obtained by ST from PT. We can observe from Fig. 4 that $\boldsymbol{E}\{R_p\} \approx \boldsymbol{E}\{R_n\}$ for the first case and the small gap between them is due to the non-zero probability of $\alpha^* = 1$. For the secondary case where $\alpha = \boldsymbol{E}\{\alpha^*_{\text{lb}}\}$ we are also able to obtain a tight lower bound for $\boldsymbol{E}\{R_n\}$. Thus the derived $\boldsymbol{E}\{\alpha^*_{\text{lb}}\}$ is able to provide us with a practical way to implement our proposed spectrum sharing protocol with minimal degradation in performance.

IV. CONCLUSIONS

We proposed a cooperative spectrum sharing scheme in which a secondary system gains spectrum access without adversely affecting the rate of the primary system. We showed that by an appropriate choice of the power allocation factor at the secondary transmitter, the secondary system is able to compensate for the interference to the primary system caused by its transmission. For the case where the direct link between the transmitter and receiver of primary system is weak, we showed that with a high probability, our proposed scheme can improve the rate of primary system. We derive threshold values for the power allocation factor such that the achievable rate of primary system can be improved as a result of the spectrum sharing protocol. Furthermore, we derived an average value of a lower bound of the power allocation factor which can be employed in practice for the proposed protocol using only statistical channel knowledge.

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